1.

$$\lim_{m\to\infty} \sqrt[m]{m^m + m!} \cdot \frac{16m^3 + 100m^2}{100m^3 + m^4} =$$

$$= \lim_{m\to\infty} m \cdot \sqrt[m]{1 + \frac{m!}{m^m}} \cdot \frac{m^3\left(16 + \frac{100}{m}\right)}{m^4\left(\frac{100}{m} + 1\right)} =$$

$$= \lim_{m\to\infty} \sqrt[m]{1 + \frac{m!}{m^m}} \cdot \lim \frac{16 + \frac{100}{m}}{\frac{100}{m} + 1} = 1 \cdot \frac{16+0}{0+1} = 16$$

$$1 \le \sqrt[m]{1 + \frac{m!}{m^m}} \le \sqrt[m]{1+1} = \sqrt[m]{2}$$

$$\begin{array}{ccccccc} \downarrow & \Rightarrow & \downarrow & \Leftarrow & & & \downarrow \\ 1 & & 1 & \text{LO2P} & & & 1 \end{array}$$

2.

$$\lim_{m\to\infty} \left(\sqrt{2m^5 - m^2 + 1} - \sqrt{2m^5 + m^2}\right)\sqrt[3]{m^2} =$$

$$= \lim \frac{\cancel{2}\, m^{\frac{2}{3}} \cdot \left(-2m^2 + 1\right)}{m^{\frac{5}{2}}\left(\sqrt{2 - \frac{1}{m^3} + \frac{1}{m^5}} + \sqrt{2 + \frac{1}{m^3}}\right)} =$$

$$= \lim m^{2 + \frac{2}{3} - \frac{5}{2}} \cdot \frac{-2 + \frac{1}{m^2}}{\sqrt{} + \sqrt{}} =$$

$$= \lim m^{\frac{1}{6}} \cdot \lim \frac{-2 + \frac{1}{m^2}}{\sqrt{} + \sqrt{}} = \infty \cdot \frac{-2+0}{\sqrt{2} + \sqrt{2}} = -\infty$$

③ $$\lim_{n\to\infty} \frac{e^n + n^5 \cdot \sin n + 5^n}{3^{n+1} + \log(3^{2^n}) + \log n!} \cdot \left(\frac{3n+3}{5n}\right)^n =$$

$$= \lim_{n\to\infty} \frac{5^n \cdot \left(\left(\frac{e}{5}\right)^n + \frac{n^5}{5^n}\sin n + 1\right)}{3\cdot 3^n + 2^n \cdot \log 3 + \log n!} \cdot \left(\frac{3}{5}\right)^n \left(\frac{n+1}{n}\right)^n =$$

$$= \lim_{n\to\infty} \frac{1 + \left(\frac{e}{5}\right)^n + \frac{n^5}{5^n}\sin n}{3^n\left(3 + \log 3 \cdot \left(\frac{2}{3}\right)^n + \frac{\log n!}{3^n}\right)} \cdot 3^n \cdot \left(1+\frac{1}{n}\right)^n$$

$$= \lim_{n\to\infty} \frac{1 + \left(\frac{e}{5}\right)^n + \frac{n^5}{5^n}\sin n}{3 + \log 3 \cdot \left(\frac{2}{3}\right)^n + \frac{\log n!}{3^n}} \cdot \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n =$$

$$= e \cdot \frac{1+0+0}{3+0+0} = \frac{e}{3}$$

$\frac{n^5}{5^n} \to 0 \quad \Rightarrow \quad \frac{n^5}{5^n}\sin n \to 0$ („nulová · omezená")

$\log n! \leq \log n^n = n \cdot \log n$, takže

$$\frac{\log n!}{3^n} \leq \frac{n \cdot \log n}{(\sqrt{3})^n \cdot (\sqrt{3})^n} = \frac{n}{(\sqrt{3})^n} \cdot \frac{\log n}{(\sqrt{3})^n} \xrightarrow{\text{srov. škála}} 0 \cdot 0$$